** BALLAJURA COMMUNITY COLLEGE**

**Year 12 Applications SAMPLE**

**Test 4 – Time series and Finance**

Calculator Assumed

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_Time: 55 min Marks: \_\_\_\_\_\_ /5 5

1. [1,1,1,2,2 : 7 marks]  
     
   An annuity compounds interest annually and its value after withdrawals can be modelled using the recurrence relation

a) Use the relation to state

i) the annual percentage interest rate.

|  |
| --- |
| **Solution** |
|  |
| **Specific behavours** |
| ✓ correct value |

ii) the initial value of the annuity.

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

b) Calculate the balance of the annuity, to the nearest dollar, after withdrawals.

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

c) The annuity is closed after withdrawals. Calculate the total interest paid by the annuity up to this time.

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates correct method   correct amount |

d) From the outset, the annual withdrawal can be reduced so that the annuity becomes a perpetuity. Briefly explain what a perpetuity is and determine the withdrawal required.

|  |
| --- |
| **Solution** |
| A perpetuity is an annuity in which the withdrawal equals the interest paid each time period and so the withdrawal can be made for ever.  Withdrawal will be |
| **Specific behaviours** |
| ✓ satisfactory explanation of perpetuity   correct new withdrawal |

2. [1,2, 3: 6 marks]

The deseasonalised number of working holiday makers in Australia over the four-year period from March 2014 to December 2017 was modelled by , where is the number of people in thousands and is the quarter, with corresponding to March 2014.

The Seasonal Index table is shown below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Quarter | March | June | September | December |
| Seasonal Index |  |  |  |  |

a) Determine the value of in the table above.

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

b) State, with reasons, whether the number of working holiday makers in Australia was highest during September 2017 or during September 2014.

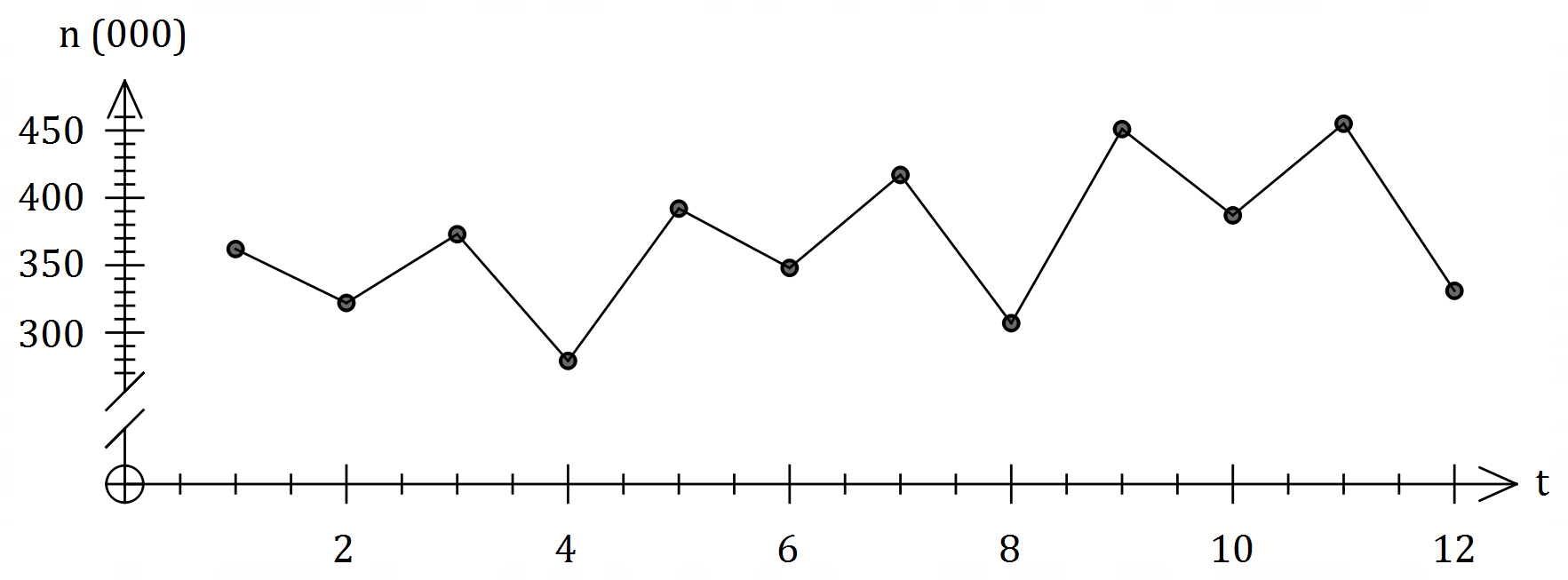
|  |
| --- |
| **Solution** |
| Sept 2014, as the gradient of the linear model is , indicating a decrease of 1431 people per quarter. |
| **Specific behaviours** |
| ✓ chooses correct year   indicates gradient implies decrease over time |

c) Use your knowledge of time series to estimate the actual change in the number of working holiday makers in Australia from December 2017 ( to March 2018 (.

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct prediction for in 's   correct prediction for in 's   correct change, adjusting for 's |

3. [3, 2, 2, 3 : 10 marks]

The number of people (, in thousands) studying in Australia with a student visa from to are shown in the graph and table below.



|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Year | Quarter | Time () | Number ( 's) | Quarterly mean | Percentage of quarterly mean |
|  |  |  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  | C |  |
|  |  |  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

a) Determine the value of , the value of and the value of in the table above.

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ value of   value of   value of |

b) Complete the missing values in the seasonal index table below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Quarter |  |  |  |  |
| Seasonal Index |  |  | 1.126 |  |

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ one correct value   both correct values |

c) Calculate the deseasonalised number of people studying in Australia with a student visa in the third quarter of .

|  |
| --- |
| **Solution** |
| Number of people is |
| **Specific behaviours** |
| ✓ indicates correct calculation   correct figure, allowing for thousands |

Let represent all the deseasonalised values of . The equation of the least-squares line for against is given by .

d) Determine an estimate for the number of people studying in Australia with a student visa in the fourth quarter of if the existing trend and seasonality continues.

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses   correct value of   correct estimate, allowing for thousands |

4. [2, 2, 3 : 7 marks]

A reducing balance loan is defined by the recurrence relation , where is the balance of the loan in dollars at the start of month . The relation was used to create the following spreadsheet.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Month | Balance of loan at start of month | Monthly interest | Monthly repayment | Loan balance carried forward |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

a) Determine the annual percentage interest rate that applies to the loan.

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ calculates monthly rate   correct annual rate as percentage |

b) State the value of each of the constants and in the recurrence relation.

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ value of   value of and value of |

c) Determine the value of , the value of and the value of shown in the spreadsheet.

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ value of  ✓ value of  ✓ value of |

5. [3, 2, 2 : 7 marks]

On March Sam started a new job with an annual salary of . At that time, the balance of his superannuation fund from previous jobs was . Sam's new employer deposits a sum equal to of his monthly salary into his fund on the last day of each month.

Interest on the balance of an individual's superannuation fund is added on the last day of each month, just before any deposits are made, and the fund advertises an interest rate of per annum.

a) Determine the balance of Sam's superannuation fund on April .

|  |
| --- |
| **Solution** |
| Interest multiplier: .  Deposit: .  New balance: |
| **Specific behaviours** |
| ✓ interest multiplier (or interest amount)   deposit   correct balance |

b) Write a recursive relation for the balance of Sam's superannuation fund months after she started her new job.

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ recursive part   initial term |

c) Calculate the expected increase in the balance of Sam's superannuation fund after he has been in his new job for one year and if his circumstances do not change.

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct balance after one year   correct increase |

6. [1, 2, 2, 2, 1, 2 : 10 marks]

A factory operates three consecutive eight-hour shifts A, B and C each day. The table below shows the number of workers who turned up late for each shift, together with a three-point moving average .

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Time period | Day | Shift | Number late | Moving average |
|  | Sun | A |  |  |
|  | Sun | B |  | 41 |
|  | Sun | C |  |  |
|  | Mon | A |  |  |
|  | Mon | B |  |  |
|  | Mon | C |  |  |
|  | Tue | A |  |  |
|  | Tue | B |  |  |
|  | Tue | C |  |  |

a) Briefly describe the purpose of calculating a set of moving averages for a time series.

|  |
| --- |
| **Solution** |
| To smooth time series data and expose the underlying trend. |
| **Specific behaviours** |
| ✓ indicates smoothing |

b) Determine the value of and the value of in the table above.

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ value of   value of |

c) Determine the centred six-point moving average for .

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates appropriate method to centre   correct average |

d) Determine the least-squares line to predict from .

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ slope, to at least dp   intercept, to at least dp  *NB Using CAS, first entry for must be .* |

Two of the seasonal indices for the above time series are shown in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
| Shift | A | B | C |
| Seasonal index |  |  | 1.03 |

e) Calculate the seasonal index for shift B.

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ value of index |

f) Forecast the number of late workers for the next C shift (on Thursday), using the  
least-squares line from (d) and making any necessary seasonal adjustment.

|  |
| --- |
| **Solution** |
| Forecast late workers. |
| **Specific behaviours** |
| ✓ value using least-squares line   correct forecast, as whole number |

7. [2, 3, 2 : 7 marks]

A person has a credit card account with an outstanding debt of and the card provider charges interest at a rate of per annum compounded daily.

a) Determine their card debt in days' time if the card is not used for any more purchases and no repayments are made.

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates method (possibly a financial calculator)   correct debt |

The person can pay off their card debt using an unsecured loan from their bank at an interest rate of compounded monthly.

b) Use effective interest rates to determine, with reasoning, whether the unsecured loan would be a better option for this person.

|  |
| --- |
| **Solution** |
| Card:  Loan:  The loan is not a better option as the effective interest rate is higher. |
| **Specific behaviours** |
| ✓ one correct rate (to at least dp)   both correct rates   explains why loan is not a better option |

The person chose to pay off their card debt in full by taking out a two year secured loan for from a lender who compounds interest quarterly. At the end of this time, the person must repay the principal and interest, a sum of .

c) Determine the interest rate charged by this lender.

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates method (possibly a financial calculator)   correct rate |